Quiz 8

February 17, 2017

Show all work and circle your final answer.

1. Write the *form* of the partial fraction decomposition of

$$\frac{x^3+1}{x^2+2x}.$$

Do *not* determine the numerical value of the coefficients.

$$\begin{array}{r} x-2 \\ x^2+2x\,\overline{)\,x^3+0x^2+0x+1} \\ -(x^3+2x^2) \\ \hline -2x^2\quad +1 \\ -(-2x^2-4x) \\ \hline 4x+1 \end{array}$$

$$\frac{x^3+1}{x^2+2x} = x-2+\frac{4x+1}{x^2+2x}$$

$$= x-2+\frac{4x+1}{x(x+2)}$$

$$= \boxed{x-2+\frac{A}{x}+\frac{B}{x+2}}$$

2. Evaluate $\displaystyle\int \frac{x^2-x+6}{x^3+3x}\,dx.$ $= \displaystyle\int \frac{2}{x}+\frac{-x-1}{x^2+3}\,dx$

$$\frac{x^2-x+6}{x(x^2+3)} = \frac{A}{x}+\frac{Bx+C}{x^2+3}$$

$x^2-x+6 = A(x^2+3)+(Bx+C)x$

$x=0$: $6=3A \longrightarrow \boxed{A=2}$

$x^2-x+6 = Ax^2+3A+Bx^2+Cx$

$x^2-x+6 = (2+B)x^2+Cx+6$ (since $A=2$)

$2+B=1 \longrightarrow \boxed{B=-1}$, $\boxed{C=-1}$

$$= 2\ln|x| + \int \frac{-x}{x^2+3}+\frac{-1}{x^2+3}\,dx$$

$$= \boxed{2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}}+C}$$

using $u=x^2+3$

since $\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

3. Make a substitution to express

$$\int_0^1 \frac{e^t}{(e^t-2)(e^{2t}+1)}\,dt$$

as a rational function. Do not evaluate the integral.

The reason this isn't a rational function is that there are e^t's.

$$\int_0^1 \frac{e^t}{(e^t-2)(e^{2t}+1)}\,dt$$

$u=e^t$

$du=e^t\,dt$

Notice: $e^{2t}=(e^t)^2=u^2$

$$= \int_{e^0}^{e^1} \frac{1}{(u-2)(u^2+1)}\,du = \boxed{\int_1^e \frac{1}{(u-2)(u^2+1)}\,du}$$